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[This question paper contains 3 printed pages.]

5/12/17

Your Roll No..............

Sr. No. of Question Paper : 5761 H

Unique Paper Code : 222101

Name of the Paper : Mathematical Physics – I (PHHT-101)

Name of the Course : B.Sc. (Hons.) PHYSICS

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all including Q. No. **1** which is compulsory.

1. Do any *five* of the following : $5 \times 3 = 15$

(a) Are the following vectors linearly dependent or independent:
$\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}, \vec{B} = \hat{\imath} + 2\hat{\jmath}, \vec{C} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

(b) Find the unit vectors perpendicular to the plane $6x + 2y + 3z = 6$. Draw the section of the plane in the first octant.

(c) Find the derivative of $f(x, y, z) = x^2 - xy^2 - xe^z$ at P_0 (1,1,0) in the direction of $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.

(d) Evaluate $\iint \vec{r}\cdot\hat{n}\ ds$ over the surface S of a sphere of radius A.

(e) Find $J\left(\frac{x,y}{u,v}\right)$, given $x = u^2 - v^2$ and $y = 2uv$.

P.T.O.

(f) What is the period of the function: $f(t) = 3\sin 2\pi t + \cos \pi t$.

(g) Evaluate $\int_0^\infty \sqrt{y}\, e^{-y^3}\, dy$

(h) Graph the following function and classify it as odd, even or neither.

$$f(x) = \begin{cases} x^2 & -\pi < x < 0 \\ 0 & 0 < x < \pi \end{cases}$$

2. a) If $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal. 5

b) Prove that: $\vec{\nabla} f(r) = \frac{df}{dr}\frac{\vec{r}}{r}$. 3

c) Define conservative force field. Show that the following field $\vec{F}$ is conservative and find the scalar potential for it. Find the work done in moving a particle in this field from (0,0,1) to (2,0,3).

$$\vec{F} = (2xz + 2xy)\,\hat{\imath} + (x^2 - y)\,\hat{\jmath} + (2z + x^2)\,\hat{k}.$$ 7

3. a) Evaluate $\iint_s \vec{A}.\hat{n}\, ds$; $\vec{A} = (x + y^2)\hat{\imath} - 2x\,\hat{\jmath} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. 4

b) State Green's Theorem. Show that if region R is bounded by simple closed curve C, then area of region R is given by: $\frac{1}{2}\oint_c xdy - ydx$. 4

c) Verify Stokes' Theorem for $\vec{A} = (2x - y)\hat{\imath} - yz^2\,\hat{\jmath} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 7

4. a) Derive expression for the Gradient of a scalar field in an orthogonal curvilinear coordinates. Give its expression for cylindrical coordinates. 4

b) Prove that $J\left(\frac{x,y,z}{u,v,w}\right) J\left(\frac{u,v,w}{x,y,z}\right) = 1$. 3

c) Evaluate $\iiint_v (x^2 + y^2 + z^2)\, dx\, dy\, dz$, where V is a sphere having center at the origin and radius equal to A. 4

d) Prove that the cylindrical co-ordinate system is orthogonal. 4

5. a) State the Dirichlet theorem for Fourier series expansion of a function. 3

b) Expand $f(x) = e^x$ in a cosine series over the interval (0, 1) 7

c) Expand the following periodic function in Fourier series

$$f(x) = \begin{cases} x + 1 & -1 < x < 0 \\ x - 1 & 0 < x < 1 \end{cases}$$ 5

6. a) Give Fourier Series for the following periodic function

$f(x) = x \sin x \qquad -\pi < x < \pi$.

And hence deduce that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\times 3} - \frac{1}{3\times 5} + \frac{1}{5\times 7} - \cdots$ 8

b) Find the Fourier series expansion of output of a half wave rectifier. Draw the rectified function between 0 and T, where T is the time period. 7

7. a) Prove that

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$ 4

b) State and prove Normal Law of errors. 7

c) Show that $\int_0^1 x^m\, (lnx)^n\, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ 4

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[This question paper contains 6 printed pages.]

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Your Roll No..............

Sr. No. of Question Paper : 5762 **H**

Unique Paper Code : 222103

Name of the Paper : PHHT-102 Mechanics

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Five** questions in all including Q. No. **1** which is compulsory.

1. Attempt any **five** of the following :

(a) A satellite of mass m is in a circular orbit of radius r around the Earth. What is its angular momentum?

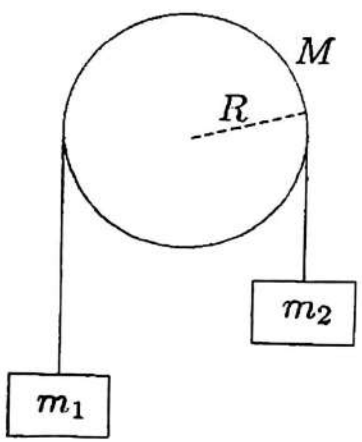
(b) A 500 gram weight is suspended below a vertical wire of length 2 meters and radius 0.2 mm. If it produces an extension of 1.0 cm in the wire, what is the Young's Modulus of the material of the wire?

(c) A rubber ball of mass 100 g falls from a height of 1 meter. It bounces off the floor and rises back to almost its original height. The time of contact between the ball and ground is 1 millisecond. Find the impulse and force on the ball due to the ground.

(d) Verify that the radius vector sweeps out equal areas in equal intervals of time for any elliptical orbit under central force motion.

(e) The shearing stress across a cross-section of water flowing through a tube of diameter 2 cm is 10^{-2} N m^{-2} and the viscosity of water is approximately 10^{-3} N m^{-2} s. Find the difference in water velocity between the center and the edge of the tube.

(f) Two masses m_1 and m_2 are connected by a massless string and hang on either side of a massive pulley of mass M and radius R attached to a fixed support. Find the resultant acceleration of the two masses.

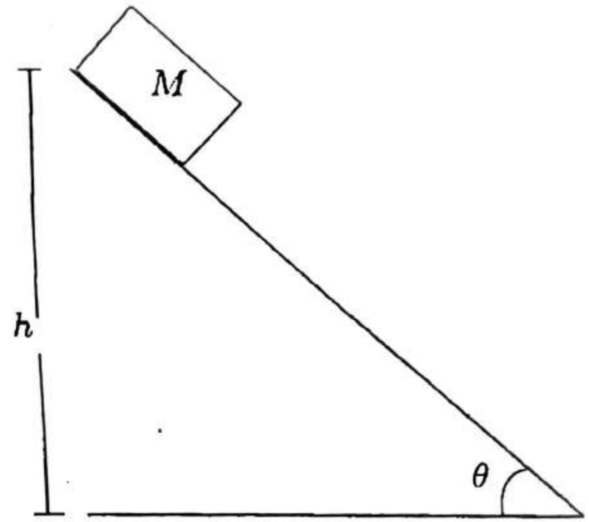


(g) State the relativistic formula for the Doppler effect for light when the observer is at an angle θ from the line of motion. What will be the ratio of observed frequency when the source moves directly towards observer to the observed frequency when source moves directly away?

(h) What will be the ratio of the rotation period of a Foucault pendulum suspended at latitude 60 degrees North to one suspended at 30 degrees South? (5×3=15)

2. (a) Derive the Work-Energy Theorem in three dimensions clearly explaining all assumptions. Define potential energy in terms of force and re-write the Work-Energy Theorem in terms of potential energy. Derive the potential energy function for a repulsive inverse-square force. Draw the energy diagram for this potential and describe the features of motion in this potential. (5,2,2,1)

(b) A block of mass M slides down an irtclined plane of angle θ. Find the speed of the block after it has descended a vertical height h using Work-Energy Theorem. Assume it starts at rest and coefficient of friction is μ between the block and the plane. (5)



3. (a) State and prove the Parallel Axis Theorem for rigid body rotation drawing a clear diagram arid explaining all steps. (6)

(b) Derive the moment of inertia I of a uniform disc of mass M and radius R about an axis passing through its center and perpendicular to its plane. (6)

(c) Now use the parallel axis theorem to find the moment of inertia of the same disc about another axis passing half-way between its center and its edge, parallel to the axis through its center. (3)

4. (a) Define gravitation potential. Derive the expression for the gravitational potential due to a spherical shell of radius R and mass M at a point outside the shell and also at a point inside the shell. (10)

(b) Sketch the graph of both gravitational potential and the gravitational field as a function of radial distance from the center of the shell showing both inside and outside region. (5)

5. (a) Derive an expression for the twisting couple per unit twist for a solid cylinder as well as a hollow one. Show that a hollow cylinder is stronger than a solid one of same material, mass and length. (5,3)

(b) Derive an expression relating the elastic constants Y (Young's Modulus) and K (Bulk Modulus) to σ (Poisson's ratio) (7)

6. (a) What are Einstein's postulates of relativity? Use them to derive the Lorentz transformations. (8)

(b) What is Coriolis force? Show that the total Coriolis force acting on a body of mass m in a rotating frame is $-2\, m\, \omega \times v_r$, where ω is the angular velocity of rotating frame and v_r is the velocity of the body in rotating frame. (7)

7. (a) Derive the relativistic law of velocity transformations. Use this to show that the speed of light is measured to be the same in all inertial frames of reference. (8)

(b) Using Lorentz transformations, deduce the time-dilation formula. Unstable muon particles have a mean lifetime τ in the laboratory of 2.15×10^{-6} seconds. Cosmic ray muons travel at a spéed very close to the velocity of light. Assuming they are observed to move at four-fifth the speed of light, what will be the observed life-time? (3,4)

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This question paper contains 4+2 printed pages] 4/12/17

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6671**

Unique Paper Code : **32221101** **HC**

Name of the Paper : **Mathematical Physics—I**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) Two sides of a triangle are formed by the vectors :

$$\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k} \text{ and } \vec{B} = 4\hat{i} - \hat{j} + 3\hat{k}$$

Determine the angle between these two sides and length of the third side.

P.T.O.

(*b*) Show that the area bounded by a simple closed curve C is given by :

$$\frac{1}{2}\oint_C (x\,dy - y\,dx).$$

(*c*) If $\vec{a}$ is a constant vector, then prove that :

$$\vec{\nabla} \times \left(\vec{a} \times \vec{r}\right) = 2\vec{a}.$$

(*d*) Solve :

$$\iint_R \sqrt{x^2 + y^2}\,dx\,dy$$

where, R is the region bounded by the circle, $x^2 + y^2 = 9$.

(*e*) Check whether the following functions are linearly independent or not :

$$e^x,\ x\,e^x.$$

(*f*) Solve the differential equation :

$$(b^2 + 2xy + y^2)dx + (x + y)^2 dy = 0.$$

(*g*) Form a differential equation whose solution is given by :

$$y = A\,e^{2x} + B\,e^{3x}.$$

(*h*) Solve :

(*i*) $\displaystyle\int_0^5 \delta(x - \pi)\cos 2x\,dx$

(*ii*) $\displaystyle\int_{-2}^{2} \left[x^2 + \log x\right]\delta(x - 1)\,dx.$

2. (*a*) Find the constants '*a*' and '*b*' so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2). 4

(*b*) If $\vec{A} = r^n\,\vec{r}$, then find the value of *n* for which $\vec{A}$ is solenoidal. 5

(*c*) Prove that : 6

$$\vec{\nabla}\cdot\left[r\,\vec{\nabla}\left(\frac{1}{r^3}\right)\right] = \frac{3}{r^4}$$

where, $r = \sqrt{x^2 + y^2 + z^2}$.

3. (a) Prove that : 6

$$\vec{A} \times \left(\vec{\nabla} \times \vec{A}\right) = \frac{1}{2}\vec{\nabla} A^2 - \left(\vec{A}.\vec{\nabla}\right)\vec{A}.$$

(b) Evaluate $\iint_S \left(\vec{A}.\hat{n}\right) dS$, where :

$$\vec{A} = y\hat{i} + 2x\hat{j} - z\hat{k}$$

And,

S is the surface of the plane, $2x + y = 6$ in the first octant cut-off by the plane, $z = 4$. 9

4. (a) Prove that : 5

$$\oiint_S r^5 \hat{n}\, dS = \iiint_V 5r^3 \vec{r}\, dV$$

where, simple closed surface S encloses volume V.

(b) Write the mathematical form of Gauss's Divergence theorem and hence verify it for $\vec{F} = 4xz\,\hat{i} - y^2\hat{j} + yz\,\hat{k}$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 1,9

5. (a) Evaluate :

$$\iiint_V (2x + y)\, dV,$$

where, V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes, $x = 0$, $y = 0$, $y = 2$, $z = 0$. 6

(b) Derive an expression for curl of a vector field in orthogonal curvilinear coordinates. Express it in cylindrical coordinates. 7,2

6. Solve the differential equations :

(a) $(x^2y - 2xy^2)dx - (x^3 - 2x^2y)dy = 0$ 7

(b) $(D^2 + 1)y = \text{cosec}\, x \quad \left(D = \dfrac{d}{dx}\right)$. 8

7. (a) Solve the differential equation : 7

$$(D^2 - 6D + 8)y = (e^{2x} - 1)^2.$$

(b) Using method of variation of parameters, solve the differential equation : 8

$$(D^2 + 4)y = x \sin 2x.$$

8. (*a*) Solve the differential equation : 7

$$(D^2 - 4D + 3)y = xe^{2x}.$$

(*b*) Using method of undetermined coefficients, solve the differential equation : 8

$$(D^2 - 1)y = e^x + 2x.$$

This question paper contains 4+2 printed pages]

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6672**

Unique Paper Code : **32221102** **HC**

Name of the Paper : **Mechanics**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. Attempt any *five* of the following questions :

 (*i*) Locate the centre of mass of a system of three particles masses 1 kg, 2 kg and 3 kg placed at the corners of an equilateral triangle of side 1 m.

P.T.O.

(*ii*) When a particle moves under a central force, prove that the particle moves in a fixed plane.

(*iii*) Show that damping has little or no effect on the frequency of a harmonic oscillator if its quality factor is large.

(*iv*) State Kepler's laws of planetary motion.

(*v*) Show that a conservative force can be expressed as $\vec{F} = -\nabla V$, where V is the potential energy.

(*vi*) What is potential energy curve ? Identify stable, unstable and neutral equilibrium from the curve.

(*vii*) Calculate the recessional velocity of a galaxy at a distance of 3×10^9 light years. Is this velocity relativistic ?

(*viii*) Explain the physical significance of negative results obtained from Michelson-Morley experiment. 3×5=15

2. (*a*) Find the centre of mass of a homogeneous semi-circular plate of radius R. 3

(*b*) An object falling in the earth's gravitational field gains mass from surrounding stationary material :

(*i*) Show that :

$$M\frac{dv}{dt} + v\frac{dM}{dt} = Mg,$$

where v is the instantaneous downward velocity of the object when its mass is M. 2

(*ii*) If :

$$\frac{dM}{dt} = kM,$$

where *k* is a constant, show that the object acquires a terminal velocity and determine this velocity. 4

(*c*) A particle is projected with a velocity of 40 m/s at an elevation of 30°. Calculate (*i*) the greatest height attained (*ii*) the horizontal range and (*iii*) the velocity and direction at a height of 12 m. 2,2,2

3. (*a*) Show that in the case of an elastic and glancing collision between two particles of masses m_1 and m_2 respectively, the maximum value of scattering angle θ_1 in the

laboratory frame corresponds to the scattering angle θ in the centre mass reference frame, where $\theta = \cos^{-1}(-m_2/m_1)$.

Also show that this maximum value of the scattering angle $\theta_1 = \tan^{-1}\left(\frac{m_1^2}{m_2^2} - 1\right)^{1/2}$. 5,2

(*b*) Show that if a heavy particle is incident on a light particle initially at rest, the heavy particle will not bounce backward as a result of collision. 3

(*c*) Prove that in centre of mass system, the magnitude of the velocities of the particles remains unaltered in elastic collision. 5

4. (*a*) Find the moment of inertia of a uniform solid cylinder of radius R, height H and mass M about an axis passing through its centre of mass and perpendicular to its axis of symmetry. 8

(*b*) Show that the ratio of rotational to translational kinetic energy for a solid cylinder rolling down a plane without slipping is 1 : 2. 3

(*c*) Moment of inertia of a bigger solid sphere about its diameter is I. 64 smaller, equal spheres are made out of bigger sphere. What will be the moment of inertia of such smaller sphere about its diameter ? 4

5. (*a*) Derive the expressions for gravitational field and potential at a point inside and outside a uniform solid sphere of radius R and mass M. 5,5

(*b*) Represent the variations of field and potential graphically with respect to distance from the centre of the shell. 5

6. (*a*) Derive differential equation for a forced harmonic oscillator and find its steady state solution. Obtain the amplitude and phase of the steady state solution. 2,6,2,2

(*b*) What is the displacement of a particle executing SHM from its mean position when its KE is half of its PE ? 3

7. (*a*) What is longitudinal and transverse Doppler effect in light. Obtain an expression for the apparent frequency of light pulse in case of longitudinal Doppler effect in a moving frame of reference. 2,9

(*b*) How does mass of a material particle change with velocity ? Show that c is the ultimate speed of the particle in free space. 2,2

8. (*a*) Derive the relativistic law of variation of mass with velocity. For a relativistic particle, show that : 7,3

$$E^2 = p^2c^2 + m_0^2c^4.$$

(*b*) Find the velocity that an electron must be given so that its momentum is 10 times its rest mass times the speed of light. What is the energy at this speed ?

(Rest mass of electron = 9×10^{-31} kg). 3,2